Varying Cosmological Constant and the Machian Solution in the Generalized Scalar-Tensor Theory of Gravitation

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Abstract

The cosmological constant $(1/2)\lambda_1\phi_{,\mu}\phi^{,\mu}/\phi^2$ is introduced to the generalized scalar-tensor theory of gravitation with the coupling function $\omega(\phi) = \eta/(\xi-2)$ and the Machian cosmological solution satisfying $\phi = O(\rho/\omega)$ is discussed for the homogeneous and isotropic universe with a perfect fluid (with negative pressure). We need the closed model and the negative coupling function for the attractive gravitational force. The constraint $\omega(\phi) < -3/2$ for $0 \leqq \xi < 2$ leads to $\eta > 3$. If $\lambda_1 < 0$ and $0 \leqq -\eta/\lambda_1 < 2$, the universe shows the slowly accelerating expansion. The coupling function diverges to $-\infty$ and the gravitational constant converges to G_∞ when $\xi \to 2$ $(t \to +\infty)$. The cosmological constant decays in proportion to t^{-2}. Thus the Machian cosmological model approaches to the Friedmann universe in general relativity with $\ddot{a} = 0$, $\lambda = 0$, and $p = -\rho/3$ as $t \to +\infty$. General relativity is locally valid enough at present.

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Since the generalized scalar-tensor theory of gravitation [1]-[3] was proposed, many attempts to determine the arbitrary coupling function $\omega(\phi)$ have been made (see, for example, [4]-[9] and references therein). Recently we discussed a new aspect of the coupling function in the Machian point of view and proposed $\omega(\phi) = \eta/(\xi - 2)$ [10]. This coupling function does not include explicitly the scalar field ϕ but depends on the parameter ξ, and varies in time very slowly owing to the physical revolution of matter in the universe $(0 \leqq \xi(t) < 2)$. As the parameter ξ approaches to 2 (negative pressure $\gamma = -1/3$), the coupling function diverges to the minus infinity. If we assume $\epsilon \equiv 2-\xi \sim 10^{-3}$ at present, we obtain $\omega \sim -10^3$ which is compatible with the recent observations [11].

For this coupling function, the generalized scalar-tensor theory of gravitation has a simple cosmological solution satisfying $\phi = O(\rho/\omega)$ (Machian solution) for the homogeneous and isotropic universe with a perfect fluid with pressure. The scalar field in this closed model converges to a finite constant $G_\infty^{-1} > 0$ (, which does not differ much from the present gravitational constant G_0^{-1}) when $\xi \to 2$ (probably, $t \to +\infty$). So the Machian cosmological solution realize dynamically the almost constant gravitational "constant" as the result of the evolution of the universe.

However, this cosmological model shows the extremely slowly decelerating expansion when $\xi \to 2$, which is not compatible with the recent measurements [12] of the distances to type Ia supernovae. In the present paper, we discuss the Machian cosmological solution in the (modified) generalized scalar-tensor theory of gravitation with the varying cosmological constant to realize the slowly accelerating expansion of the universe.

Let us start with the following action [1]-[3]

$$S = \int d^4x\sqrt{-g}\left\{-\phi\left[R + 2\lambda(\phi, \phi_{,\mu}\phi^{,\mu})\right] + 16\pi L_m - \frac{\omega(\phi)}{\phi}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right\}, \tag{1}$$

where R is the scalar curvature of the metric $g_{\mu\nu}$, $\phi(x)$ is the Brans-Dicke scalar field, $\omega(\phi)$ is an arbitrary coupling function, and L_m represents the Lagrangian for the matter fields. The term $\lambda(\phi, \phi_{,\mu}\phi^{,\mu})$ represents the cosmological constant, which is modified from $\lambda(\phi)$, a function of only ϕ. It should be noted that the term $\phi_{,\mu}\phi^{,\mu}$ is a scalar. The ordinary cosmological constant must be positive $(\lambda > 0)$ for the cosmological repulsion. We assume a particular form of the cosmological constant in the Machian point of view (in order to realize a solution satisfying $\phi = O(\rho/\omega)$):

$$\lambda(\phi, \phi_{,\mu}\phi^{,\mu}) \equiv \frac{1}{2}\lambda_1\phi_{,\mu}\phi^{,\mu}/\phi^2, \tag{2}$$

where λ_1 is a constant (later, we require $\lambda_1 < 0$). For this particular cosmological constant, we obtain from the action Eq.(1)

$$S = \int d^4x\sqrt{-g}\left\{-\phi R + 16\pi L_m - \frac{[\omega(\phi)+\lambda_1]}{\phi} g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right\}. \tag{3}$$

We can regard that the present cosmological constant is a constant-part of the coupling function of ϕ. The variation of Eq.(3) with respect to $g_{\mu\nu}$ and ϕ leads to the field equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi}{\phi}T_{\mu\nu} - \frac{[\omega(\phi)+\lambda_1]}{\phi^2}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,\lambda}\phi^{,\lambda}\right) - \frac{1}{\phi}(\phi_{,\mu;\nu} - g_{\mu\nu}\Box\phi), \tag{4}$$

$$\Box\phi = -\frac{1}{3+2[\omega(\phi)+\lambda_1]}\left[8\pi T + \frac{d\omega(\phi)}{d\phi}\phi_{,\lambda}\phi^{,\lambda}\right], \tag{5}$$

which satisfy the conservation law of the energy-momentum $T_{\mu\nu}$

$$T^{\mu\nu}_{;\nu} = 0. \tag{6}$$

The line element for the Friedmann-Robertson-Walker metric is

$$ds^2 = -dt^2 + a^2(t)[d\chi^2 + \sigma^2(\chi)(d\theta^2 + \sin^2\theta d\varphi^2)], \tag{7}$$

where $\sigma(\chi)$ is $\sin\chi$, χ, and $\sinh\chi$ for closed ($k=+1$), flat ($k=0$), and open ($k=-1$) spaces, respectively. The energy-momentum tensor for the perfect fluid with pressure p and the mass density ρ is given as

$$T_{\mu\nu} = -pg_{\mu\nu} - (\rho+p)u_\mu u_\nu. \tag{8}$$

The nonvanishing components are $T_{00} = -\rho$, $T_{ii} = -pg_{ii}$ ($i \neq 0$), and its trace is $T = \rho - 3p$ for the homogeneous and isotropic universe.

The energy conservation Eq.(6) leads to the equation of continuity

$$\dot\rho + 3\frac{\dot a}{a}(\rho + p) = 0, \tag{9}$$

which gives, with the barotropic equation of state

$$p(t) = \gamma\rho(t), \; -1 \leqq \gamma \leqq 1/3, \tag{10}$$

$$\rho(t)a^n(t) = const, \tag{11}$$

where $n = 3(\gamma+1)$.

The nonvanishing components of the field equation (4) are

$$2a\ddot{a} + \dot{a}^2 + k = -\frac{[\omega(\phi) + \lambda_1]}{2} a^2 \left(\frac{\dot{\phi}}{\phi}\right)^2 + a\dot{a}\left(\frac{\dot{\phi}}{\phi}\right) - \frac{8\pi a^2 p}{\phi} - \frac{8\pi a^2 (\rho - 3p)}{3 + 2[\omega(\phi) + \lambda_1]} \frac{1}{\phi}, \tag{12}$$

and

$$\frac{3}{a^2}\left(\dot{a}^2 + k\right) = \frac{[\omega(\phi) + \lambda_1]}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\ddot{\phi}}{\phi} + \frac{8\pi\rho}{\phi} - \frac{8\pi(\rho - 3p)}{3 + 2[\omega(\phi) + \lambda_1]}\frac{1}{\phi}. \tag{13}$$

The field equation (5) gives

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = \frac{1}{3 + 2[\omega(\phi) + \lambda_1]}\left[8\pi(\rho - 3p) - \frac{d\omega(\phi)}{d\phi}\dot{\phi}^2\right]. \tag{14}$$

We adopt as usual Eqs.(13), (14), and (11) as the independent equations to solve simultaneously.

Let us require as the coupling function

$$\omega(\phi) \equiv \frac{\eta}{\xi - 2}, \tag{15}$$

which is necessarily derived from the condition for the reasonable Machian solution [10]. The parameter $\xi = 1 - 3\gamma$ varies in time very slowly as a quasi-static process and so we may regard that the parameter ξ is constant when we execute the derivative with respect to t. We introduce another scalar function $\Phi(t)$ by

$$\phi(t) = \frac{8\pi}{3 + 2[\omega(\phi) + \lambda_1]}\Phi(t) \tag{16}$$

for the Machian solution satisfying $\phi = O(\rho/\omega)$. Taking Eq.(16) and $d\omega/d\phi = 0$ into account, we obtain from Eq.(14)

$$\ddot{\Phi} + 3\frac{\dot{a}}{a}\dot{\Phi} = \xi\rho, \tag{17}$$

which means that the ratio $\dot{a}/a$ does not include ω, and so we find for the expansion parameter

$$a(t) \equiv A(\omega)\alpha(t), \tag{18}$$

where A and α are arbitrary functions of only ω and t respectively.

After eliminating $\ddot{\phi}$ by Eq.(14), we get from Eq.(13)

$$\frac{\omega}{2}\left[\left(\frac{\dot{\Phi}}{\Phi}\right)^2+\frac{4\rho}{\Phi}\right]-\frac{3k}{A^2(\omega)\alpha^2}$$

$$= 3\left(\frac{\dot{\alpha}}{\alpha}\right)^2+3\left(\frac{\dot{\alpha}}{\alpha}\right)\left(\frac{\dot{\Phi}}{\Phi}\right)-\frac{\lambda_1}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2-\frac{(3+2\lambda_1)\,\rho}{\Phi}. \tag{19}$$

For the closed and the open spaces ($k=\pm 1$), if we require that Eq.(19) is identically satisfied for all arbitrary values of ω, we find that the coefficient $A(\omega)$ must have the following form

$$\frac{3}{A^2(\omega)}=\left|\frac{\omega(\phi)}{2}+B\right|, \tag{20}$$

where B is a constant with no dependence of ω and furthermore we obtain

$$\left(\frac{\dot{\Phi}}{\Phi}\right)^2+\frac{4\rho}{\Phi}\equiv k\,j\frac{1}{\alpha^2} \tag{21}$$

and

$$3\left(\frac{\dot{\alpha}}{\alpha}\right)^2+3\left(\frac{\dot{\alpha}}{\alpha}\right)\left(\frac{\dot{\Phi}}{\Phi}\right)-\frac{\lambda_1}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2-\frac{(3+2\lambda_1)\,\rho}{\Phi}\equiv -k\,j\frac{B}{\alpha^2} \tag{22}$$

using a notation $j=-1$ for $\omega/2+B<0$ and $j=+1$ for $\omega/2+B>0$.

Thus we find the similar Machian cosmological solution in the generalized scalar-tensor theory of gravitation with the varying cosmological constant,

$$\Phi(t)=\zeta\rho(t)t^2 \tag{23}$$

and

$$\alpha(t)=bt \tag{24}$$

with

$$\zeta=1/(\xi-2)\,, \tag{25}$$

$$b=\begin{cases}(4-\xi^2)^{-1/2}, & for\ k\,j=-1\ and\ 0\leqq\xi<2\\ (\xi^2-4)^{-1/2}, & for\ k\,j=+1\ and\ 2<\xi\leqq 4\,,\end{cases} \tag{26}$$

and

$$B=\frac{\lambda_1}{2}-\frac{3}{(\xi-2)(\xi+2)}\,. \tag{27}$$

There is a discontinuity at $\xi = 2$ and we restrict to the range $0 \leqq \xi < 2$ owing to the evolutionary continuity from $\xi = 0$ (the radiation era) and $\xi = 1$ (the dust-dominated era). So we require the closed model ($k = +1$, $j = -1$) for the attractive gravitational force ($G > 0$). Both the coupling function $\omega(\phi) = \eta/(\xi - 2)$ for $0 \leqq \xi < 2$ and the constraint

$$\frac{\omega(\phi)}{2} + B < 0 \tag{28}$$

lead necessarily to $\eta > 0$ and $\omega < 0$. We require $\eta > 3$ to avoid the singularity ($\omega < -3/2$). We get $\omega(\phi) = -\eta/2$ and $B = \lambda_1/2 + 3/4$ when $\xi = 0$, and so the constraint Eq.(28) gives $\eta - 3 > 2\lambda_1$. Experimentally, if we find $|\omega| \sim 10^3$ [11], we obtain $2 - \xi \equiv \epsilon \sim 10^{-3}$ (assuming $\eta \approx 3$). Though the constants ζ and b diverge at $\xi = 2$, the scalar field ϕ and the expansion parameter $a(t)$ themselves do not diverge at $\xi = 2$. The crucial point at $\xi = 2$ is that the sign of the coupling function $\omega(\phi)$ reverses there ($j = -1 \to j = +1$ for $k = +1$).

The expansion parameter is finally expressed as

$$a(t) \equiv A(\omega)bt = [6/f(\xi)]^{1/2}\, t\,, \tag{29}$$

where

$$f(\xi) \equiv \lambda_1(\xi - 2)(\xi + 2) + \eta(\xi + 2) - 6\,. \tag{30}$$

When $\xi \to 2$, we get the finite expansion parameter

$$a(t) = \sqrt{3/(2\eta - 3)}\, t\,. \tag{31}$$

For the enough expansion at present, we need require that the constant η is not so much. The value $\eta = 3$ gives the expansion parameter $a(t) = t$ (light velocity). If $\eta > 3$ and $\lambda_1 < 0$, the quadratic function $f(\xi)$ is always positive for the variable ξ ($0 \leqq \xi < 2$) and satisfies Eq.(28). The function $f(\xi)$ becomes a maximum at $\xi_{\max} = -\eta/\lambda_1$. In the case $\xi_{\max} \geqq 2$, $f(\xi)$ gives a monotonous increasing function for $0 \leqq \xi < 2$, and in the case $0 < \xi_{\max} < 2$, gives a monotonous decreasing function for $\xi_{\max} \leqq \xi < 2$. If $\eta > 3$ and $\lambda_1 > 0$, the quadratic function $f(\xi)$ is positive for $0 \leqq \xi < 2$ when $\eta - 3 > 2\lambda_1$, and gives always a monotonous increasing function for $0 \leqq \xi < 2$. If $\lambda_1 = 0$, then the function $f(\xi)$ becomes a linear function and shows a monotonous increase for $\eta > 0$. Therefore, if $\lambda_1 < 0$ and $0 \leqq -\eta/\lambda_1 < 2$, the universe shows the slowly accelerating expansion for the period $-\eta/\lambda_1 \leqq \xi < 2$.

The scalar field ϕ for the coupling function $\omega(\phi) = \eta/(\xi - 2)$ is given as the following and we obtain when $\xi \to 2$

$$\phi(t) = \frac{8\pi\rho(t)t^2}{(3 + \lambda_1)(\xi - 2) + 2\eta} \cong \frac{4\pi\rho(t)t^2}{\eta} \to const\,, \tag{32}$$

which converges to a definite and finite constant in the limit. The asymptotic behavior ($\xi \to 2$) of the scalar field is the same as that of the case without the cosmological constant. It should be remarked that the gravitational constant G is determined by the mass density and the age of the universe if we adopt $\eta \approx 3$ as the coupling function is large enough. If $t_0 = 1.5 \times 10^{10}\, yr$, we obtain $\rho_0 = 1.6 \times 10^{-29}\, g.cm^{-3}$, which is very near to the critical density $\rho_c \sim 10^{-29}\, g.cm^{-3}$. As the parameter $\xi \to 2$, the coupling function $\omega(\phi)$ diverges to the minus infinity and the gravitational constant approaches dynamically to the constant G_∞

The cosmological constant $\lambda(t)$ decreases rapidly in proportion to t^{-2} as the universe expands and converges to zero when $\xi \to 2$ ($t \to +\infty$):

$$\lambda(t) = \frac{\lambda_1}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 \propto \frac{(\xi-2)^2}{t^2} \to 0\,. \tag{33}$$

The effective cosmological constant $\Lambda(t)$ introduced in the previous paper [13] also decreases rapidly and converges to zero when $\xi \to 2$ ($t \to +\infty$):

$$\Lambda(t) \equiv -\frac{\omega(\phi)}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 \propto \frac{\xi-2}{t^2} \to 0\,. \tag{34}$$

It should be noted that the sign of the cosmological term $\lambda(t) < 0$ is opposite to that of the usual cosmological term in the (i,i) components of the field equations, and inversely the effective cosmological term $\Lambda(t) > 0$ is opposite in the $(0,0)$ component.

We can estimate the order of each terms appeared in Eqs.(12) and (13) by means of the present Machian cosmological solution when $\omega(\phi) = \eta/(\xi-2) \to -\infty$, for example,

$$\frac{8\pi}{3+2\left[\omega(\phi)+\lambda_1\right]}\frac{a^2\xi\rho}{\phi} \sim \xi - 2 \to 0\,, \tag{35}$$

$$\frac{8\pi a^2\rho}{\phi} \to \frac{6\eta}{2\eta-3} \sim O(1)\,, \tag{36}$$

$$\frac{8\pi a^2 p}{\phi} \to -\frac{2\eta}{2\eta-3} \sim O(1)\,, \tag{37}$$

$$\frac{\omega(\phi)}{2}a^2\left(\frac{\dot{\phi}}{\phi}\right)^2 \sim \xi - 2 \to 0\,, \tag{38}$$

$$\frac{\lambda_1}{2}a^2\left(\frac{\dot{\phi}}{\phi}\right)^2 \sim (\xi-2)^2 \to 0\,, \tag{39}$$

$$a\dot{a}\left(\frac{\dot{\phi}}{\phi}\right) \sim \xi - 2 \rightarrow 0\,, \tag{40}$$

and

$$a^2\frac{\ddot{\phi}}{\phi} \sim \xi - 2 \rightarrow 0\,. \tag{41}$$

The crucial difference from the correspondences [14] in the Brans-Dicke theory [15] is that even the term Eq.(38) converges to zero when $\omega(\phi) \rightarrow -\infty$. This means that the abnormal term vanishes and the generalized scalar-tensor theory of gravitation reproduces the correspondent solution of general relativity with the same energy-momentum tensor completely when $\xi \rightarrow 2$ ($t \rightarrow +\infty$).

In the Brans-Dicke theory, the coupling parameter ω is arbitrary, and so we realize that the Machian solution does not reduce to that of general relativity for the fixed finite time t when $|\omega| \rightarrow +\infty$. However, we cannot determine, for example, the limit of the term $\Lambda(t) \propto \omega/t^2$ when $|\omega| \rightarrow +\infty$ and $t \rightarrow +\infty$ in the Brans-Dicke theory. If we regard the coupling function $\omega(\phi)$ as an arbitrary parameter in the generalized scalar-tensor theory, we obtain the similar correspondences (for example, $\omega(\phi)a^2\left(\dot{\phi}/\phi\right)^2 \sim O(1)$) in the Brans-Dicke theory for the same Machian cosmological solution for the fixed finite time t. In this case, the generalized scalar-tensor theory does not reduce to general relativity when $|\omega| \rightarrow +\infty$ and $\omega^{-3}d\omega/d\phi \rightarrow 0$. We find that the scalar field ϕ converges to zero when $|\omega| \rightarrow +\infty$ according to the postulate $\phi = O(\rho/\omega)$. For the present Machian cosmological solution in the generalized scalar-tensor theory, we know the behavior of the coupling function $\omega(\phi)$ when $t \rightarrow +\infty$ and so we can estimate definitely the limit of the solution when $t \rightarrow +\infty$. Thus the generalized scalar-tensor theory of gravitation approaches dynamically to general relativity in the result of the evolution of the universe when $t \rightarrow +\infty$.

Let us remember the Friedmann equations with the cosmological term for the homogeneous and isotropic universe in general relativity:

$$2a\ddot{a} + \dot{a}^2 + k - \lambda a^2 = -\kappa p a^2\,, \tag{42}$$

$$\frac{3}{a^2}\left(\dot{a}^2 + k\right) - \lambda = \kappa\rho\,, \tag{43}$$

where κ is Einstein's gravitational constant. We get from Eq.(42) when $\ddot{a} = 0$ and $p = -\rho/3$

$$\frac{3}{a^2}\left(\dot{a}^2 + k\right) - 3\lambda = \kappa\rho\,, \tag{44}$$

and find $\lambda = 0$. If we require $\ddot{a}(t) = 0$ and $\lambda(t) \to 0$ $(t \to +\infty)$, we obtain $p \to -\rho/3$. This fact supports that the final state of the Machian cosmological solution with the decaying cosmological constant is the negative pressure $\gamma = -1/3$ $(\xi = 2)$. Taking Eqs.(31) and (35)-(41) $(k = +1)$, and $\kappa \equiv 8\pi G_\infty$ into account, we observe that Eqs.(12) and (13) reduce to Eqs.(42) and (43) respectively when $\omega(\phi) \to -\infty$ $(t \to +\infty)$. The constant η still remains indefinite. It is interesting that the Machian cosmological solution in the Brans-Dicke theory, the coupling parameter of which is constant and arbitrary, is correspondent to the static Einstein universe ($\dot{a} \to 0$ when $\omega \to -\infty$).

The cosmological constant $\lambda(\phi)$ is not consistent with the Machian cosmological solution satisfying $\phi = O(\rho/\omega)$ for the variable parameter ξ. Only the present form of the cosmological constant $(1/2)\lambda_1\phi_{,\mu}\phi^{,\mu}/\phi^2$ admits the Machian solution in the generalized scalar-tensor theory of gravitation. The constraint $\omega(\phi) < -3/2$ leads to $\eta > 3$. If $\lambda_1 < 0$ and $0 \leqq -\eta/\lambda_1 < 2$, the universe surely shows the slowly accelerating expansion for the later period from $\xi = -\eta/\lambda_1$, though the expansion parameter is explicitly a linear function of t. For example, if we adopt $\eta = 3$ and $\lambda_1 = -3$, the universe exhibits the slowly decelerating expansion before the dust-dominated era $\xi = 1$ (for the positive pressure), the slowly accelerating expansion after the dust-dominated era (for the negative pressure) respectively.

In the generalized scalar-tensor theory of gravitation, not only the gravitational constant but also the variation of the coupling function is derived from the evolution of the universe. The remaining problem is to determine the time-variation of the parameter $\xi(t)$ by the physical evolution of matter in the universe. Our conjecture is that the universe passed the radiation era $(\xi = 0)$ and the dust-dominated era $(\xi = 1)$ rapidly in the early stage, and has been staying the negative pressure era ($1 < \xi < 2$, maybe almost near $\xi = 2$) for the almost all period of $10^{10}yr$. Finally, the universe will approach to the state of $\xi = 2$ $(\gamma = -1/3)$ as it expands for ever.

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