Determination of the Coupling Function in the Generalized Scalar-Tensor Theory of Gravitation

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Abstract

Some possible coupling functions $\omega(\phi)$ and their behaviors are discussed in the generalized scalar-tensor theory of gravitation in the context of the Machian cosmological model with the condition $\phi = O(\rho/\omega)$. The discussions are restricted to the homogeneous and isotropic universe with a perfect fluid (with negative pressure). We propose $\omega(\phi) = \eta/(\xi - 2)$ for the coupling function. The parameter ξ varies in time very slowly from $\xi = 0$ to $\xi = 2$ because of the physical evolution of matter in the universe. When $\xi \to 2$, the coupling function diverges to $-\infty$ and the scalar field ϕ converges to G_∞^{-1}. The present mass density is precisely predicted if the present time of the universe is given. We obtain $\rho_0 = 1.6 \times 10^{-29}\, g.cm^{-3}$ for $t_0 = 1.5 \times 10^{10}\, yr$. The universe shows the slowly decelerating expansion for the time-varying coupling function.

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In the previous paper [1], we obtained the Machian cosmological solution satisfying $\phi = O(\rho/\omega)$ for the homogeneous and isotropic universe with a perfect fluid (with *negative pressure*) in the Brans-Dicke theory [2]. We found that the gravitational constant surely approaches to constant when the coefficient γ of the equation of state goes to $-1/3$ for the closed model. If we assume that the present mass density ρ_0 is identical to the critical density ρ_c, taking $|\omega| \sim 10^3$ [3] into account, we get the difference ϵ of the coefficient γ from $-1/3$ ($\epsilon \equiv 3\gamma + 1$) has a value of order 10^{-3} to support the present gravitational constant G. These parameters lead to the time-variation of the gravitational constant $\left|\dot{G}/G\right| \sim 10^{-13}\,yr^{-1}$ which is compatible with the recent observational data [4]. Thus, it avoids the difficulty of varying gravitational constant in the Brans-Dicke theory to introduce a perfect fluid with *negative pressure*.

The closed model ($k = +1$) of this Machian solution is valid for the coupling parameter ω satisfying $\omega/2 + B < 0$, where $B \equiv -3/(\xi+2)(\xi-2)$ and $\xi \equiv 1 - 3\gamma$. The above parameter $\epsilon \sim 10^{-3}$ gives $B \sim 10^3$ which means $\omega < -10^3$ at present. This situation may explain the reason why the coupling parameter $|\omega|$ is so large at present. The expansion parameter $a(t)$ is explicitly a linear function of t, but its coefficient depends on the parameter ξ. As the parameter ξ goes to 2 through the quasi-static process, the coefficient of the expansion parameter increases very slowly for the *constant* coupling parameter ω, and so the universe exhibits the slowly accelerating expansion at present, which is also compatible with the recent measurements [5] of the distances to type Ia supernovae.

The above result ($\omega/2 + B < 0$) strongly suggests that the coupling parameter ω of the Brans-Dicke scalar field should vary in time as the universe expands. If not so, the *constant* coupling parameter ω should be infinite from the beginning of the universe. A tentative conjecture is like $\omega(\phi(t)) \sim B(\xi)$. The coupling parameter ω is almost constant and varies in time very slowly through the quasi-static process of the parameter ξ. Therefore the time-variation of the coupling parameter depends on the physical evolution of matter in the universe.

In the present paper, we discuss the coupling function $\omega(\phi)$ and its behavior in the generalized scalar-tensor theory of gravitation [6]-[8] in the context of the Machian cosmological model. We consider the homogeneous and isotropic universe filled with a perfect fluid with pressure. We require the Machian postulate that the cosmological scalar field ϕ should have the asymptotic form $\phi = O(\rho/\omega)$ for the large enough coupling function ω. The procedure developed in the preceding paper [9] gives a simple and effective method to seek the Machian cosmological solution.

The action [6]-[8] for the generalized scalar-tensor theory of gravitation is described in our sign conventions ($c = 1$) as

$$S = \int d^4x\sqrt{-g}[-\phi R + 16\pi L_m - \frac{\omega(\phi)}{\phi} g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}] , \tag{1}$$

where R is the scalar curvature of the metric $g_{\mu\nu}$, $\phi(x)$ is the Brans-Dicke scalar field, $\omega(\phi)$ is an arbitrary coupling function, and L_m represents the Lagrangian for the matter fields. The variation of Eq.(1) with respect to $g_{\mu\nu}$ and ϕ leads to the field equations

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} &= \frac{8\pi}{\phi}T_{\mu\nu} - \frac{\omega(\phi)}{\phi^2}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,\lambda}\phi^{,\lambda}\right) \\ &\quad -\frac{1}{\phi}(\phi_{,\mu;\nu} - g_{\mu\nu}\Box\phi) , \end{aligned} \tag{2}$$

$$\Box\phi = -\frac{1}{3+2\omega(\phi)}\left[8\pi T + \frac{d\omega(\phi)}{d\phi}\phi_{,\lambda}\phi^{,\lambda}\right] , \tag{3}$$

where $T_{\mu\nu}$ is the energy-momentum tensor, T is the contracted energy-momentum tensor, and $\Box$ denotes the generally-covariant d'Alembertian $\Box\phi \equiv \phi^{,\mu}{}_{;\mu}$. These field equations satisfy the conservation law of the energy-momentum

$$T^{\mu\nu}_{;\nu} = 0 . \tag{4}$$

The line element for the Friedmann-Robertson-Walker metric is

$$ds^2 = -dt^2 + a^2(t)[d\chi^2 + \sigma^2(\chi)(d\theta^2 + \sin^2\theta d\varphi^2)] , \tag{5}$$

where

$$\sigma(\chi) \equiv \begin{cases} \sin\chi & for\ k = +1\ (closed\ space) \\ \chi & for\ k = 0\ \ (flat\ space) \\ \sinh\chi & for\ k = -1\ \ (open\ space) . \end{cases} \tag{6}$$

The energy-momentum tensor for the perfect fluid with pressure p is given as

$$T_{\mu\nu} = -pg_{\mu\nu} - (\rho + p)u_\mu u_\nu , \tag{7}$$

where ρ is the mass density in comoving coordinates and u^μ is the four velocity $dx^\mu/d\tau$ (τ is the proper time). The nonvanishing components are $T_{00} = -\rho$, $T_{ii} = -pg_{ii}$ $(i \neq 0)$, and its trace is $T = \rho - 3p$ for the homogeneous and isotropic universe.

The energy conservation Eq.(4) leads to the equation of continuity

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 . \tag{8}$$

Let us suppose the equation of state for the perfect fluid

$$p(t) = \gamma \rho(t)\,, \tag{9}$$

where $-1 \leqq \gamma \leqq 1/3$. Now we are rather interested in the "negative" pressure. Taking the equation of state into account, we can integrate the equation of continuity Eq.(8) and get

$$\rho(t) a^n(t) = const\,, \tag{10}$$

where $n = 3(\gamma + 1)$, which has $n = 4$, $n = 3$, and $n = 2$ for the radiation era, the matter-dominated era, and the negative pressure $\gamma = -1/3$ era respectively.

The independent field equations which we need solve simultaneously are

$$\frac{3}{a^2}\left(\dot{a}^2 + k\right) = \frac{\omega(\phi)}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\ddot{\phi}}{\phi} + \frac{8\pi\rho}{\phi} - \frac{8\pi\left(\rho - 3p\right)}{3 + 2\omega(\phi)}\frac{1}{\phi}\,, \tag{11}$$

and

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = \frac{1}{3 + 2\omega(\phi)}\left[8\pi\left(\rho - 3p\right) - \frac{d\omega(\phi)}{d\phi}\dot{\phi}^2\right]\,. \tag{12}$$

According to the postulate $\phi = O(\rho/\omega)$, we expect a solution described as

$$\phi(t) = \frac{8\pi}{3 + 2\omega(\phi)}\Phi(t)\,, \tag{13}$$

where $\Phi(t)$ is an unknown function of t and may generally include ω as the following form

$$\Phi(t) = \Phi_0(t) + O(1/\omega)\,. \tag{14}$$

In the Brans-Dicke theory, we know that $\Phi(t)$ should not include ω for the perfect fluid model [9].

The most important thing is to determine an arbitrary coupling function $\omega(\phi)$. There are many literatures, for example [10]-[15]. We examine some possibilities in the Machian point of view. First, if the coupling function $\omega(\phi)$ obeys

$$\frac{d\omega(\phi)}{d\phi}\dot{\phi}^2 = 8\pi\eta\rho(t)\,, \tag{15}$$

we can adopt the similar procedure [9] for the Machian cosmological solution in the Brans-Dicke theory. The coefficient η is an arbitrary constant.

Let us assume that the following relation is satisfied approximately enough because the coupling function $\omega(\phi)$ is almost constant or large enough:

$$\dot{\phi}(t) = \frac{8\pi}{3 + 2\omega(\phi)}\dot{\Phi}(t)\,. \tag{16}$$

We need check later whether this simplification is valid or not for the obtained Machian solution. Taking Eq.(16) into account, we get from Eqs.(12) and (15)

$$\ddot{\Phi} + 3\frac{\dot{a}}{a}\dot{\Phi} = (\xi + \eta)\,\rho\,, \tag{17}$$

where $\xi = 1 - 3\gamma$ or $\xi = 4 - n$. The function $\Phi(t)$ should not include ω similarly in the Brans-Dicke theory for the Machian solution because the right-hand side of Eq.(15) does not depend on ω. Thus the ratio $\dot{a}/a$ should not also include ω, and so we find for the expansion parameter

$$a(t) \equiv A(\omega)\alpha(t)\,, \tag{18}$$

where A and α are arbitrary functions of only ω and t respectively.

Taking Eqs.(13), (16), and (18), we obtain from Eq.(11) after elimination of $\ddot{\phi}$ by Eq.(12)

$$\frac{\omega}{2}\left[\left(\frac{\dot{\Phi}}{\Phi}\right)^2 + \frac{4\rho}{\Phi}\right] - \frac{3k}{A^2(\omega)\alpha^2} = 3\left(\frac{\dot{\alpha}}{\alpha}\right)^2 + 3\left(\frac{\dot{\alpha}}{\alpha}\right)\left(\frac{\dot{\Phi}}{\Phi}\right) - \frac{(3+\eta)\,\rho}{\Phi}\,. \tag{19}$$

We are not interested in the flat space ($k = 0$) here. For the closed and the open spaces ($k = \pm 1$), if we require that Eq.(19) is identically satisfied for all arbitrary values of ω, we find that the coefficient $A(\omega)$ must have the following form

$$\frac{3}{A^2(\omega)} = \left|\frac{\omega(\phi)}{2} + B\right|\,, \tag{20}$$

where B is a constant with no dependence of ω. Thus we get

$$\frac{\omega}{2}\left[\left(\frac{\dot{\Phi}}{\Phi}\right)^2 + \frac{4\rho}{\Phi} - kj\frac{1}{\alpha^2}\right] = 3\left(\frac{\dot{\alpha}}{\alpha}\right)^2 + 3\left(\frac{\dot{\alpha}}{\alpha}\right)\left(\frac{\dot{\Phi}}{\Phi}\right) - \frac{(3+\eta)\,\rho}{\Phi} + kj\frac{B}{\alpha^2}\,, \tag{21}$$

where we introduce a notation $j = -1$ for $\omega/2 + B < 0$ and $j = +1$ for $\omega/2 + B > 0$. Again, by requiring that Eq.(21) is identically satisfied for all ω, we obtain two identities,

$$\left(\frac{\dot{\Phi}}{\Phi}\right)^2 + \frac{4\rho}{\Phi} \equiv kj\frac{1}{\alpha^2}\,, \tag{22}$$

and

$$3\left(\frac{\dot{\alpha}}{\alpha}\right)^2 + 3\left(\frac{\dot{\alpha}}{\alpha}\right)\left(\frac{\dot{\Phi}}{\Phi}\right) - \frac{(3+\eta)\,\rho}{\Phi} \equiv -kj\frac{B}{\alpha^2}\,. \tag{23}$$

We are convinced of the existence of the following type of solutions on the analogy of the previous paper [1]:

$$\Phi(t) = \zeta\rho(t)t^2 \,, \tag{24}$$

$$\alpha(t) = bt \,, \tag{25}$$

where coefficients ζ and b are constants respectively, and in fact we find

$$\zeta = \frac{\xi+\eta}{\xi(\xi-2)} \,, \tag{26}$$

$$b^2 = \begin{cases} -\frac{\xi+\eta}{(\xi-2)\left[\xi^2+(\eta+2)\xi-2\eta\right]} \,, & for\ kj = -1\ and\ 0 \leqq \xi < 2 \\ \frac{\xi+\eta}{(\xi-2)\left[\xi^2+(\eta+2)\xi-2\eta\right]} \,, & for\ kj = +1\ and\ 2 < \xi \leqq 4 \,, \end{cases} \tag{27}$$

and

$$B = \frac{\eta\xi^2 - (5\eta+3)\xi + 3\eta}{(\xi-2)\left[\xi^2 + (\eta+2)\xi - 2\eta\right]} \tag{28}$$

from Eqs.(17), (22), and (23) successively.

For the closed space ($k = +1$), we require $\eta > 0$ for $\zeta < 0$ when $0 \leqq \xi < 2$ to give the attractive gravitational force ($G > 0$). On the other hand we require $\eta < 0$, for the coefficient b is real when $0 \leqq \xi < 2$. So we encounter a contradiction. Let us adopt the other alternative that the coefficient η is a linear function of ξ ($\eta = \eta_1\xi + \eta_0$). We put $\eta_0 = 0$ for the finite ζ when $\xi = 0$. Thus we replace Eq.(15) to

$$\frac{d\omega(\phi)}{d\phi}\dot{\phi}^2 = 8\pi\eta_1\xi\rho(t) = 8\pi\eta_1\left(\rho - 3p\right) \,. \tag{29}$$

After similar discussions, we obtain as a solution

$$\zeta = \left(\eta_1 + 1\right)/(\xi - 2) \,, \tag{30}$$

$$b^2 = \begin{cases} -1/(\xi-2)(\xi-\xi_1) \,, & for\ kj = -1\ and\ 0 \leqq \xi < 2 \\ 1/(\xi-2)(\xi-\xi_1) \,, & for\ kj = +1\ and\ 2 < \xi \leqq 4 \,, \end{cases} \tag{31}$$

and

$$B = \left[\eta_1(\xi^2 - 5\xi + 3) - 3\right]/(\xi-2)\left[(\eta_1+1)\xi - 2(\eta_1 - 1)\right] \,, \tag{32}$$

where $\xi_1 \equiv 2(\eta_1 - 1)/\left(\eta_1 + 1\right)$.

We require $-1 < \eta_1 < 1$ for $\zeta < 0$ and the real b when $0 \leqq \xi < 2$ ($k = +1$). By analyzing the time-dependences of the solution, we get from

Eq.(29)

$$\frac{d\omega}{d\phi} = \frac{\eta_1 \xi (3+2\omega)}{(\eta_1+1)(\xi-2)} \frac{1}{\phi} . \tag{33}$$

After integration, we obtain the coupling function

$$|3+2\omega(\phi)| = C\phi(t)^{[2\eta_1\xi/(\eta_1+1)(\xi-2)]} , \tag{34}$$

where C is an integral constant. However, we realize that this coupling function $\omega(\phi)$ is not consistent with Eq.(16) because $\omega(\phi)$ varies in time too rapidly.

As the next alternative, we suppose

$$\frac{d\omega(\phi)}{d\phi}\dot{\phi}^2 = \frac{8\pi\eta\rho(t)}{3+2\omega(\phi)} \tag{35}$$

to suppress the rapid time-variation of $\omega(\phi)$. In this case, the right-hand side of Eq.(17) includes the coupling function, and so the function $\Phi(t)$ may also include $\omega(\phi)$ as Eq.(14). From now on, we restrict our calculations to the first order in $1/(3+2\omega)$. At least for the present time ($|\omega| \gtrsim 10^3$), this restriction would give a good approximation. For the first order in $1/(3+2\omega)$, the function $\Phi(t)$ does not include $\omega(\phi)$. Thus we obtain the Machian solution for the first order in $1/(3+2\omega)$ again,

$$\zeta = 1/(\xi-2) , \tag{36}$$

$$b = \begin{cases} (4-\xi^2)^{-1/2} , & for\ k\,j = -1\ and\ 0 \leqq \xi < 2 \\ (\xi^2-4)^{-1/2} , & for\ k\,j = +1\ and\ 2 < \xi \leqq 4 , \end{cases} \tag{37}$$

and

$$B = -3/(\xi-2)(\xi+2) . \tag{38}$$

In Eqs.(33) or (34), the scalar field ϕ includes the coupling function $\omega(\phi)$ itself. This expression causes ambiguities, and so let us define another function of Φ

$$\omega(\phi) = \varpi(\Phi) . \tag{39}$$

Taking this expression into account, we estimate Eq.(35) by the Machian solution for the first order in $1/(3+2\omega)$ and get

$$\frac{d\varpi}{d\Phi} = \frac{\eta}{\xi-2}\frac{1}{\Phi} , \tag{40}$$

which leads to

$$\varpi(\Phi) = \frac{\eta}{\xi-2} \ln|\Phi(t)| + W , \tag{41}$$

where W is an integral constant.

When $\xi \to 2$, the time-dependence of $\rho(t)$ goes to t^{-2} and so $\rho(t)t^2 \to$ *const*. However, the coefficient ζ diverges to the minus infinity as $\xi \to 2$ in the range of $0 \leqq \xi < 2$. So the function $\Phi(t) = \zeta\rho(t)t^2$ also diverges to the minus infinity very slowly as $\xi \to 2$ in the same range. Thus, if $\eta > 0$, the coupling function $\varpi(\Phi)$ diverges to the minus infinity when $\xi \to 2$ in this range. On the other hand, when $t \to 0$ and $\xi \to 0$, $\Phi(t)$ and $\varpi(\Phi)$ also diverge to the minus infinity. The time-derivative of the scalar field $\phi(t)$ is described in the present solution as

$$\dot{\phi}(t) = \frac{8\pi}{3+2\varpi(\Phi)}\dot{\Phi}(t)\left[1 - \frac{16}{3+2\varpi(\Phi)}\frac{\eta}{\xi-2}\right], \tag{42}$$

and the second term of the middle bracket ($\sim 1/\ln|\Phi(t)|$) goes to zero when $\xi \to 0$ and $\xi \to 2$. Therefore the assumption Eq.(16) is consistent in our Machian solution near $\xi = 0$ and $\xi = 2$. The coupling function Eq.(41) satisfies the constraint

$$\frac{\varpi(\Phi)}{2} + B < 0, \tag{43}$$

which leads to

$$\eta \ln|\Phi(t)| + (\xi - 2)W > 6/(\xi + 2), \tag{44}$$

if $\eta > 0$ in the range of $0 \leqq \xi < 2$ (at least near $\xi = 0$ and $\xi = 2$, or for the appropriate W). The behavior of the scalar field becomes

$$\phi(t) \sim (\xi - 2)\frac{\Phi(t)}{\ln|\Phi(t)|} = \frac{\rho(t)t^2}{\ln|\zeta\rho(t)t^2|} \to const \to 0 \tag{45}$$

when $\xi \to 2$. The scalar field is approaching to *const* as the parameter $\xi \to 2$ and its value gradually converges to zero.

The expansion parameter is expressed as

$$a(t) \equiv A(\omega)bt = \left[\frac{6}{\varpi(\Phi)(\xi+2)(\xi-2) - 6}\right]^{1/2} t \tag{46}$$

for the closed space ($k = +1$) in the range of $0 \leqq \xi < 2$. Taking $\varpi(\Phi) \to -\infty$ when $t \to 0$ into account, we get $a(t) \approx 0$ near $\xi = 0$. By replacing η to $\eta_1\xi$, we might be able to improve the situation, that is,

$$\frac{d\omega(\phi)}{d\phi}\dot{\phi}^2 = \frac{8\pi\eta_1(\rho - 3p)}{3 + 2\omega(\phi)}, \tag{47}$$

and thus

$$\varpi(\Phi) = \frac{\eta_1\xi}{\xi - 2}\ln|\Phi(t)| + W. \tag{48}$$

When we consider the case near $\xi = 2$, we encounter a severe trouble: the coefficient of $a(t)$ in Eq.(46) converges to zero as $\Phi(t) \to -\infty$ when $t \to \infty$. To satisfy the constraint Eq.(43), the coupling function $\varpi(\Phi)$ must decrease faster than $B \sim 1/(\xi - 2)$ when $\xi \to 2$. On the other hand, the coupling function $\varpi(\Phi)$ must decrease slower than $1/(\xi - 2)$ when $\xi \to 2$ in order to give the slowly accelerating expansion. After all, there are no alternatives but

$$\varpi(\Phi) = \frac{\eta}{\xi - 2}, \tag{49}$$

(which would give the almost linear expansion,) for the Machian cosmological solution. This means

$$\frac{d\varpi}{d\Phi} = 0. \tag{50}$$

The coupling function $\varpi(\Phi)$ does not depend on Φ. Therefore the solutions (24) and (25) with Eqs.(36), (37), and (38) are exact for all coupling parameter ω. The constraint Eq.(43) for all ξ $(0 \leqq \xi < 2)$ requires the condition $\eta > 3$, which gives $\omega < -3/2$ when $\xi = 0$ and avoids the singularity. For this coefficient $\eta > 3$, the expansion parameter $a(t)$ gives the extremely slowly decelerating expansion except the early stage with $\xi = 0$. We obtain $a(t) = t$ for $\xi \to 2$ if $\eta = 3$.

The scalar field ϕ for the coupling function $\varpi(\Phi) = \eta/(\xi - 2)$ is given as the following and we get when $\xi \to 2$

$$\phi(t) = \frac{8\pi\rho(t)t^2}{3(\xi - 2) + 2\eta} \cong \frac{4\pi\rho(t)t^2}{\eta} \to const\,, \tag{51}$$

which converges to a definite and finite constant in the limit. If we assume $\eta = 3$, taking $t_0 = 1.0 \times 10^{10}\, yr$ and the present gravitational constant $G_0 = 6.67 \times 10^{-8}\, dyn.cm^2.g^{-1}$ into account, we can estimate the present mass density $\rho_0 = 3.5 \times 10^{-29}\, g.cm^{-3}$. If we adopt $t_0 = 1.5 \times 10^{10}\, yr$, we obtain $\rho_0 = 1.6 \times 10^{-29}\, g.cm^{-3}$, which is very near to the critical density $\rho_c \sim 10^{-29}\, g.cm^{-3}$. As the parameter $\xi \to 2$, the coupling function $\varpi(\Phi)$ diverges to the minus infinity and the gravitational constant approaches dynamically to the constant G_∞.

It should be remarked that the time-variation of the coupling function $\varpi(\Phi) = \eta/(\xi - 2)$ is derived from that of the parameter ξ. However, we have not known the details of the physical evolution of matter in the universe yet. Probably, our universe started (classically) from the Big Bang with $\xi = 0$ (the radiation era), via the dust-dominated era ($\xi = 1$), and now must be approaching to the negative pressure era (the *quintessence* era, $\gamma = -1/3$, $\xi = 2$). According to the recent measurements [3] of the coupling parameter ($|\omega| \sim 10^3$), we get $\epsilon \equiv 2 - \xi \sim 10^{-3}$ from Eq.(49). Over $10^{10}\, yr$, the state of the universe has been varying extremely slowly from $\xi = 0$ to $\epsilon \sim 10^{-3}$. This situation would solve the problems existing in the solar system and the evolution of life in the time-varying gravitational constant. At $t \sim 5 \times 10^9\, yr$, the parameter ϵ had already been small enough and the gravitational constant did not differ much from the present value.

There is a discontinuity at $\xi = 2$ for the Machian cosmological solution, and so it is not reasonable that the universe continues to evolve beyond $\xi = 2$ to the state $\xi = 4$. Our universe must be approaching to the state $\xi = 2$ for ever ($t \to +\infty$). We require that the universe is closed ($k = +1$) for this range of ξ ($0 \leqq \xi < 2$) to give the attractive gravitational force. As the parameter ξ varies in time extremely slowly (the quasi-static process) in this range, we may regard that the parameter ξ is constant when we execute the derivative with respect to t.

When we deal exactly with the time-varying coupling function $\varpi(\Phi) = \eta/(\xi - 2)$, the expansion parameter $a(t)$ does not show the slowly accelerating expansion but rather the extremely slowly decelerating expansion (almost the linear expansion). If this is not compatible with the recent observations, we might need to introduce furthermore the cosmological constant to the present Machian cosmological model.

We have no criterions to determine the coupling function $\omega(\phi)$ in the generalized scalar-tensor theory of gravitation. We discussed some examples here. They seem to suggest that it is not an arbitrary function of ϕ but another scalar field which is derived from matter itself by another field equation. The scalar field ϕ was introduced to the Brans-Dicke theory through the relation $GM/c^2R \sim 1$. By the similar analogy, the relation $\omega \sim 1/(\xi - 2)$ seems to require the existence of another unknown scalar field connected with matter.

The next straightforward problem is to determine the time-variation and the range of the parameter $\xi(t)$. We need investigate the evolution of a scalar field as dark matter in the universe.

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